On the Formation of Plastic Flow Areas under Thermomechanical Effect

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Thermal stresses have a significant effect on parts of various mechanisms operating under high temperature gradients. Nonstationary temperature field variations result in the formation of residual strains and stresses. Accounting of such strains and stresses is necessary for accurate determination of the geometry and strength characteristics of the related objects. It is well known that temperature affects the yield stress of material by increasing the probability of appearance of irreversible deformations. A detailed analysis of the stress-strain state (SSS) of cylindrical bodies under a steady-state thermal action and external pressure was carried out in [1]. The observations in [2] suggest evidence of formation of irreversible deformation regions in a finite-size solid cylinder with a heat source inside it. The paper [3] compares a numerical solution of the problem on the insertion of disks with distinct temperatures using the von Mises yield condition with an analytical solution using the Tresca yield condition.

The present report studies the formation of 2D thermal stress fields when heating a clamped thin plate by a uniform temperature field. This physical process is mathematically described as a quasistatic process of uniform thermal expansion of a rectangular plate made of a thermoelastoplastic material. The generalized Prandtl–Reuss thermoelastoplastic framework is used. Two opposite sides of the plate are clamped, which ensures the yield stress state. The yield condition is taken in the von Mises form with yield stress depending on temperature. The principle of maximum dissipation energy suggests the von Mises loading surface as a plastic potential and guarantees the associated plastic flow law. Thus, the boundary value problem is reduced to systems of partial differential equations for the stress-strain state parameters. Then the resulting system is numerically integrated. A finite-difference scheme is developed. The boundaries of the irreversible deformation domain and the unloading domain are computed according to the numerical results. The residual stress and strain levels after the final cooling of the plate are calculated.

Consider the two-dimensional problem on the deformation of a thin rectangular plate in the framework of infinitesimal thermoelastoplasticity. The upper and lower sides of the plate are clamped, and the lateral sides are free from external pressure (Fig. 1). The evolution of the stress-strain state of the plate is affected by the uniform increase in the temperature field. The temperature gradient is sufficiently large to cause a plastic flow.

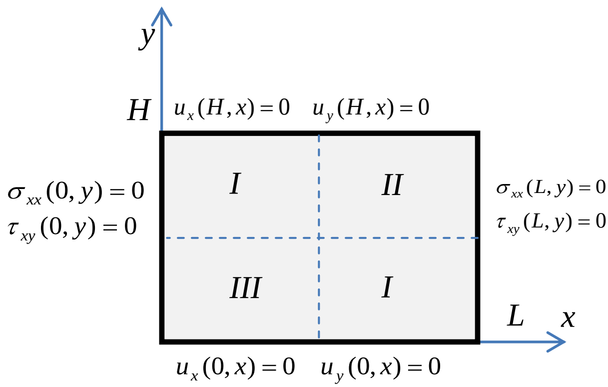

Fig. 1. The boundary conditions

The analysis in [2] of the stress-strain state of the plate shows that, regardless of the temperature, the values of some displacement and stress components along the midlines indicated in the figure remain unchanged,

$$u_x(0.5H,x)=0 \qquad \tau_{xy}(0.5H,x)=0$$
$$u_y(x,0.5L)=0 \qquad \tau_{xy}(x,0.5L)=0 \tag{1}$$

The boundary conditions (1) completely determine the boundary value problem for each quarter of the plate. Obviously, the computation time needed to solve the problem by the 2D finite difference method [3] is significantly reduced by the fourfold reduction of the number of grid points.

After calculating the displacements u_i^I and the stresses σ_{ij}^I in the upper left quarter of the plate, we find the displacement and stress fields in the remaining parts of the plate by the following formulas (where the superscript indicates the area number in accordance with Fig. 1):

$$\begin{array}{lll}
u_x^{II}(x,y) = -u_x^I(L-x,y) & u_x^{III}(x,y) = u_x^I(x,H-y) & u_x^{IV}(x,y) = -u_x^I(L-x,H-y) \\
u_y^{II}(x,y) = u_y^I(L-x,y) & u_y^{III}(x,y) = -u_y^I(x,H-y) & u_y^{IV}(x,y) = u_y^I(L-x,H-y) \\
\sigma_{xx}^{II}(x,y) = \sigma_{xx}^I(L-x,y) & \sigma_{xx}^{III}(x,y) = \sigma_{xx}^I(x,H-y) & \sigma_{xx}^{IV}(x,y) = \sigma_{xx}^I(L-x,H-y) \\
\sigma_{yy}^{II}(x,y) = \sigma_{yy}^I(L-x,y) & \sigma_{yy}^{III}(x,y) = \sigma_{yy}^I(x,H-y) & \sigma_{yy}^{IV}(x,y) = \sigma_{yy}^I(L-x,H-y) \\
\sigma_{xy}^{II}(x,y) = -\sigma_{xy}^I(L-x,y) & \sigma_{xy}^{III}(x,y) = -\sigma_{xy}^I(x,H-y) & \sigma_{xy}^{IV}(x,y) = \sigma_{xy}^I(L-x,H-y) \\
x \in [0.5L, L], \quad y \in [0.5H, H] & x \in [0, 0.5L], \quad y \in [0, 0.5H] & x \in [0.5L, L], \quad y \in [0, 0.5H]
\end{array}$$

The SSS parameters precisely coincide with those computed from the similar plate problem solved earlier. However, as was shown by the results of a numerical experiment, the calculation time was decreased by a factor of 3.

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References

[1] *D. R. Bland.* Elastoplastic Thick-Walled Tubes of Work-Hardening Material Subject to Internal and External Pressures and to Temperature Gradients. Journal of the Mechanics and Physics of Solids. Vol. 4, pp. 209–229. 1956.

[2] *Y. Orcan and U. Gamer.* Elastic–Plastic deformation of centrally heated cylinder. Acta Mechanica. Vol. 90, Issues 1–4, pp. 61–80. 1991.

[3] *Kovacs.* Residual Stresses in Thermally Loaded Shrink Fits. Periodica Polytechnica. Ser. Mech. Eng. Vol. 40. No. 2. pp. 103–112. 1996.

[4] *A. V Manzhirov, N. K. Gupta, and D. A. Indeitsev (Eds),* Topical Problems in Solid and Fluid Mechanics. Elite Publishing House, New Delhi, 2011. 411p.