Numerical Simulation of Axisymetric Oscillatory Flow in a Semi-Transparent Czochralski Oxide Melt

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Oxide single crystals are conventionally grown by the Czochralski method (Cz) primarily characterized by hydrodynamics of the melt which is inextricably coupled to transport phenomena in the fluid. The intensity and interactions of different kinds of the fluid motion determine the flow structure and heat transport in Cz melt. In a semitransparent material, such as gadolinium gallium garnet (GGG), however the heat transfer process becomes additionally complicated due to nonlinear influence of volumetric radiation on the thermal field. Therefore, semitransparent fluids behave differently than opaque ones. In the present modeling of Cz/GGG melt, the effect of internal radiative heat transfer (RHT) on the transition of fluid flow modes was studied. In this hydrodynamic model, we consider an axisymmetric flow in the melt of cylindrical geometry (r_c = 60 mm; $h_c = h_l$ = 120 mm). The crucible wall is at a constant temperature $T_w = T_{mp} + \Delta T_{max}$, and it is assumed to be an opaque gray surface (ε_c = 0.5) diffusely emitting and reflecting thermal radiation. The melt absorbs and emits but does not scatter radiation, and its optical properties (a_l (m^{-1}) and n_l) are independent of temperature and wavelength with a meniscus configuration in the vicinity of the crystal dummy (r_x = 24 mm). The melt curved free surface is a semitransparent diffuse-gray surface for which the transmissivities $\tau_{ext} = 1 - \rho_{ext} = 0.85$ and $\tau_{int} = \tau_{ext} / n^2_l = 0.23$ were estimated from Spuckler–Siegel equations for n_l = 1.938. Energy is transferred by radiation through the melt surface to radiatively black ambient walls at a constant temperature $T_a = T_{mp} - \Delta T_{max}$. The crystal is simulated by a rotating plane surface ($T_x = T_{mp}$; $\varepsilon_x = 1$) , and it is considered to have two different shapes, i.e., inclined toward the melt (model M1) and flat plate (model M2). Depending on the optical thickness ($a'_l = a_l .r_c$) of the fluid and the crystal rotation rate Ω(rad/s), strictly periodic flow was numericaly observed in the melt characterized by Pr = 4.69, $Gr = (1.142\text{x}10^3)\Delta T_{max}$, $Ma = (1.682\text{x}10^2)\Delta T_{max}$ and Re = 81.36 Ω (rad/s), where ΔT_{max} = 72K is the driving temperature difference. In the present finite volume calculations, discrete-ordinates method (with N_e x N_Φ = 25) was used to estimate the radiative flux which appears in the equations of energy. In the transient solutions, the applied time step of Δt = 0.5 s is approximately equal to ¼ of the buoyancy time scale of the model. For the melt of smaller optical thickness, $a'_l = 6 \times 10^{-3}$ the transition from steady flow mode to oscillatory one occured even for a moderate rotation rate corresponding to $Gr / \mathrm{Re}^2 = 5.52$. In this case ($Ma = 0$; Ψ_{max} = 6.3 g/s), the descending cold plume (originated at the tri-junction point) was drifted toward the center and ensuing down-ward motion along the symmetry axis. The period of oscillation t_p is equal to 14.5 s. Such a phenomenon requires the existence of some unstable stratification under the crystal due to the relatively much stranger buoyancy flow, which tends to put the hotter fluid above the cold one. In this case ($a'_l = 6 \times 10^{-3}$; $Gr / \mathrm{Re}^2 = 5.52$), thermocapillary flow does not affect the period of oscillation. In the convex-front model M1 with $a'_l = 6 \times 10^{-3}$ ($Ma = 0$), it was shown that increasing the rotation rate ($1.5 \le \Omega(rad / s) \le 3.5$), the period of oscillatory flow decreases from t_p ($Gr / Re^2 = 5.52$)~20s to t_p ($Gr / Re^2 = 1$)~12s, respectively. The period of oscillation was found to depend on the front shape as well. That is, the transient solutions of the two models M1 (convex) and M2 (flat) with the same conditions ($a'_l = 6 \times 10^{-3}$, $\Omega = 1.5$ and $Ma = 0$) lead to $t_{p,1} \cong 20s$ and $t_{p,2} \cong 14.5s$. Similar front-shape effect on the period of oscillation appeared in the melt of higher absorption coefficient ($a'_l = 6 \times 10^{-2}$, $\Omega = 1.5$ and $Ma = 0$) and may be explained by the fact that the stronger the front deviates from flat toward the melt, the weaker the flow caused by the crystal rotation. The front-shape effect on t_p found to be vanishing for the rotation rates lager than $\Omega = 1.5 rad / s$. In the case with $a'_l = 6 \times 10^{-3}$ ($Ma = 0$), the two models M1 and M2 calculations lead nearly the same period of oscillation ($t_{p,2} \cong 14.5s$) when the rotation rate was increased to $\Omega = 2.5 rad / s$ ($Gr / Re^2 = 1$). It is remarkable is that, in the convex-front model M1, there is no regime indicating the ocsillatory flow when the optical thickness is relatively large ($a'_l = 6 \times 10^{-2}$) and the crystal rotates with higher angular velocity ($2.5 \le \Omega(rad / s) \le 3.5$). This might be technically important that, the M1 model calculations with the larger driving temperature difference ($\Delta T_{max} = 108K$) result in oscillatory flow ($t_p = 12 \pm 0.5s$) for exactly the same conditions. However, in the melt of a relatively small a'_l, the period of oscillation found to be almost independent of ΔT_{max} within the applied range. In fact, as the optical thickness of the melt decreases, the directional behavior of the radiation appears more and more efficiently and so RHT tends to uniform the thermal field.