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Multibody Contact Problems for Discretely Growing Systems

K. Kazakov[1], S. Kurdina[2], and A. Manzhirov[1,2,3]

[1] Bauman Moscow State Technical University, 2nd Baumanskaya Str. 5 Bldg 1, 105005, Moscow, Russia, kazakov@ipmnet.ru

[2] Institute for Problems in Mechanics of the Russian Academy of Sciences, Vernadskogo Ave 101 Bldg 1, 119526, Moscow, Russia, manzh@inbox.ru

[3] National Research Nuclear University MEPhI, Kashirskoe Highway 31, 115409, Moscow, Russia

We study plane problems on the interaction of surface-inhomogeneous ageing viscoelastic bases with regular finite systems of rigid flat dies (see Fig. 1). The problems to be studied have the following peculiarities. First, rigid elements or groups of elements are not mounted or removed simultaneously because of special requirements observed when assembling or manufacturing engineering structures or machine parts [1–3]. Second, one has to take into account the age and structure inhomogeneity of deformable solids due to additive manufacturing processes. Such inhomogeneities can be described by rapidly oscillating and even discontinuous functions. The surface inhomogeneity of a coating usually results from a process in which the coating is continuously applied to the main layer and from surface treatment of already applied coatings (laser treatment, ion implantation, etc.). Surface inhomogeneity can also be caused by the use of various materials when manufacturing the coating.

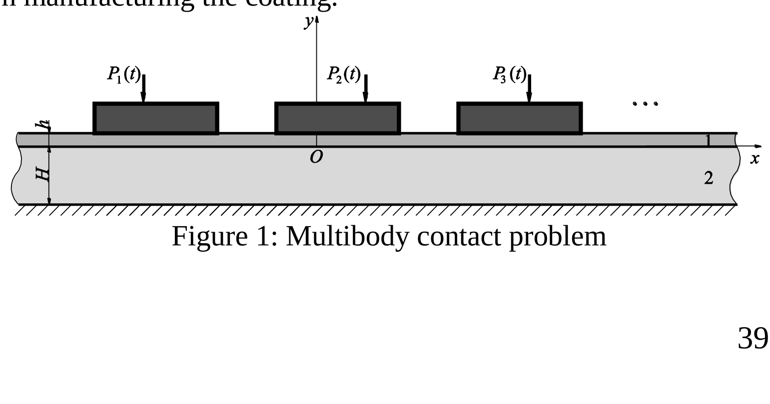

Figure 1: Multibody contact problem

The class of problems in question arises in particular when dealing with successive erection of structure complexes on bases and foundations, when determining the admissible roll angle of objects erected in close proximity to each other, or when gradually reinforcing engineering or civil engineering structures.

A general statement of the problem is given. We derive a system of resolving mixed integral equations, which is reduced in a vector function space to a single operator equation with tensor kernel and with vector additional conditions

$$c(t)m(x)\mathbf{q}(x,t)-(\mathsf{I}-\mathsf{V})\mathbf{G}\mathbf{q}(x,t)=\boldsymbol{\delta}(t)+\boldsymbol{\alpha}(t)x,$$

$$\int_{-1}^{1}\mathbf{q}(\xi,t)d\xi=\mathbf{P}(t),\quad \int_{-1}^{1}\mathbf{q}(\xi,t)\xi d\xi=\mathbf{M}(t),$$

$$\mathsf{V}\mathbf{q}(x,t)=\int_{1}^{t}K(t,\tau)\mathbf{q}(x,\tau)d\tau,\quad \mathbf{G}\mathbf{q}(x,t)=\int_{-1}^{1}\mathbf{G}_{\mathrm{pl}}(x,\xi)\mathbf{q}(\xi,t)d\xi,$$

where $\mathbf{P}(t)=P^{i}(t)\mathbf{i}^{i}$, $\mathbf{M}(t)=M^{i}(t)\mathbf{i}^{i}$, and $c(t)$ are given vector functions and scalar function of time, $m(x)$ is a given rapidly oscillating function determined by the rigidity function of the coating, $\mathbf{q}(x,t)=q^{i}(x,t)\mathbf{i}^{i}$, $\boldsymbol{\delta}(t)=\delta^{i}(t)\mathbf{i}^{i}$, and $\boldsymbol{\alpha}(t)=\alpha^{i}(t)\mathbf{i}^{i}$ are vector functions to be determined, I is the identity operator, V is a Volterra operator with given kernel $K(t,\tau)$, and $\mathbf{G}$ is a Fredholm operator with given matrix kernel $\mathbf{G}_{\mathrm{pl}}(x,\xi)$.

We can represent these equations as

$$c(t)\mathbf{Q}(x,t)-(\mathsf{I}-\mathsf{V})\mathbf{A}\,\mathbf{Q}(x,t)=\frac{\boldsymbol{\delta}(t)+\boldsymbol{\alpha}(t)x}{\sqrt{m(x)}}=\mathbf{w}(x,t),$$

$$\int_{-1}^{1}\frac{\mathbf{Q}(\xi,t)}{\sqrt{m(\xi)}}d\xi=\mathbf{P}(t),\quad \int_{-1}^{1}\frac{\mathbf{Q}(\xi,t)\xi}{\sqrt{m(\xi)}}d\xi=\mathbf{M}(t),$$

$$\mathbf{A}\,\mathbf{Q}(x,t)=\int_{-1}^{1}\mathbf{A}_{\mathrm{pl}}(x,\xi)\mathbf{Q}(\xi,t)d\xi$$

where $\mathbf{A}$ is a Fredholm operator with matrix kernel

$\mathbf{A}_{\text{pl}}(x,\xi)=\mathbf{G}_{\text{pl}}(x,\xi)\big/\sqrt{m(x)m(\xi)}$ and $\mathbf{Q}(x,t)=\sqrt{m(x)}\mathbf{q}(x,t)$ is new vector function to be determined.

We show that there exist 15 distinct versions of the statement of the problem. For various versions of the statement of the problem, we obtain analytical solutions in the Hilbert space $L_2([-1,1],V)$ on the basis of the generalized projection method [4]. To this end, we first construct an orthonormal system $\mathbf{p}_k^i(x)=p_k(x)\mathbf{i}^i$ of vector functions (basis in $L_2([-1,1],V)$), which contain the factor $1\big/\sqrt{m(x)}$.

For example, if the force $\mathbf{P}(t)$ and the moment $\mathbf{M}(t)$ of the applied load are known, then the contact pressures $\mathbf{q}(x,t)$ under each punch, the penetration depths $\boldsymbol{\delta}(t)$, and the tilt angles $\boldsymbol{\alpha}(t)$ are to be found. Then the Hilbert space $L_2([-1,1],V)$ can be represented as the direct sum of orthogonal subspaces $L_2([-1,1],V)=L_2^{(0)}([-1,1],V)\oplus L_2^{(1)}([-1,1],V)$, where $L_2^{(0)}([-1,1],V)$ is the Euclidean space with basis $\{\mathbf{p}_0^i(x),\mathbf{p}_1^i(x)\}$ ($i=1,2,\dots,n$), and $L_2^{(1)}([-1,1],V)$ is the Hilbert space with basis $\{\mathbf{p}_k^i(x)\}$ ($i=1,2,\dots,n$, $k=2,3,4,\dots$). The integrand in the main equation can be represented as a sum of functions in $L_2^{(0)}([-1,1],V)$ and $L_2^{(1)}([-1,1],V)$, $\mathbf{Q}(x,t)=\mathbf{Q}_0(x,t)+\mathbf{Q}_1(x,t)$. The representation of $\mathbf{Q}(x,t)$ contains the known first term $\mathbf{Q}_0(x,t)$, which is determined by the additional conditions and the term $\mathbf{Q}_1(x,t)$ is to be found.

Conversely, for the right-hand side, one should find $\mathbf{w}(x,t)\in L_2^{(0)}([-1,1],V)$. We introduce the orthogonal projection P_1 of $L_2([-1,1],V)$ onto $L_2^{(1)}([-1,1],V)$. Thus, we obtain an equation for determining $\mathbf{Q}_1(x,t)$. We need to construct its solution in the form of a series in the eigenfunctions of the operator $\mathsf{P}_1\mathsf{A}$. In the resulting contact stress formulas, we have managed to find closed-form expressions for the rapidly oscillating functions describing the coating rigidity, i.e., find the fine structure of the solution. Namely,

$$\mathbf{q}(x,t) = \frac{1}{m(x)}[\mathbf{z}_0(t)\, p_0^*(x) + \mathbf{z}_1(t)\, p_1^*(x) + \cdots]\,,$$

where the $\mathbf{z}_i(t)$ are vector-functions of time t and the $p_i^*(x)$ are continuous functions of the coordinate x. This permits one to obtain efficient solutions of contact interaction problems for coated foundations in which the punch base shape is described by complicated functions, in particular, by rapidly oscillating functions.

Then we introduce orthogonal projection $\mathbf{P}_0 = \mathbf{I} - \mathbf{P}_1$ of $L_2([-1,1],V)$ onto $L_2^{(0)}([-1,1],V)$ and obtain a system of algebraic equations for the penetration depths $\boldsymbol{\delta}(t)$ and the tilt angles $\boldsymbol{\alpha}(t)$.

We perform numerical calculations and make qualitative conclusions.

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References

[1] *A. V. Manzhirov*, Contact problems of the interaction between visco-elastic foundations subjected to ageing and systems of stamps not applied simultaneously, Journal of Applied Mathematics and Mechanics. Vol. 51. No. 4, pp. 523–535. 1987.

[2] *A. V. Manzhirov and V. A. Chernysh,* The contact problem of the discrete fitting of an inhomogeneous viscoelastic ageing cylinder with a system of rigid collars, Journal of Applied Mathematics and Mechanics. Vol. 55. No. 6, pp. 901–909. 1991.

[3] *N. Kh. Arutyunyan and A. V. Manzhirov*. Contact Problems in the Theory of Creep [in Russian], Erevan: Izd-vo NAN RA, 1999. 320 p.

[4] *A. V. Manzhirov, K. E. Kazakov, and S. Kucharsky*. Conformal contact between a system of punches and a covered foundation taking into account the actual shape of their surfaces, in *N. K. Gupta, A. V. Manzhirov, and R. Velmurugan (Eds)*, Topical Problems in Theoretical and Applied Mechanics, pp. 128–147. Elite Publishing House, New Delhi, 2013.