The Behavioral Pattern of the Longitudinally Vibrating Rod that is Accreting in Cross-Section Area

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This article serves as a sequel to the one authored by Shatalov M, Manzhirov A, and Fedotov I on the theory of accreting bodies entitled “On the resonant behavior of longitudinally vibrating accreting rods”. In this article, the authors analyzed the qualitative properties of the rods that were growing in length while subjected to longitudinal vibrations. The present article extends this analysis to the cantilevered rod that is subjected to longitudinal vibrations as its cross-section area grows proportionally to time, while the length of the rod is kept constant. The vibrating rod is assumed to be elastic and isotropic. The analysis is described using the linear classical model (wave equation), the Rayleigh–Love model, and the Rayleigh–Bishop model. In these analyses, it is further assumed that both the cross-section area and the polar moment of inertia are functions of time and hence variable. It is shown that irrespective of the method used for the solution of the stated resulting problem, the amplitudes of vibration in all these models decrease with time. In the classical model, it is shown that the amplitudes of vibration of the rod decrease with time for $\alpha>0$, an arbitrary real integer or half-integer. That is, as the cross-section area increases with time, the amplitudes of vibration are decreasing proportionally to time. The phenomenon is reversed for $\alpha<0$. In this particular case, the phenomenon exhibits resonance.

In both the Rayleigh–Love and Rayleigh–Bishop models, using the Galerkin–Kantorovich method for the solution of the stated problem, it is shown that there is a marked decrease in the amplitudes of vibration as time progresses, consistent with what has been observed in the classical model.

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