Fundamentals of Continuous Growth Processes in Technology and Nature

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There are two main ways of continuous growth in technology and nature known to researchers. These are volumetric and surface growth processes. We consider the latter case; for the former, we make a reference to the monographs [1, 2], where fundamental results concerning volumetric growth are presented.

A vast majority of objects or solids that surround us arise from some surface growth processes. As an example, one can present many technologies in industry, including well-known technologies of crystal growth, laser deposition, solidification of melts, electrolytic formation, pyrolytic deposition, polymerization, concreting, and modern digital additive manufacturing technologies. Similar processes determine the specific features of natural phenomena such as the growth of biological tissues, glaciers, blocks of sedimentary and volcanic rocks, space objects, etc.

Recent research has shown that solids formed by growth processes differ in their properties essentially from solids in the traditional view. Moreover, the classical approaches of solid mechanics fail to model the behavior of growing solids . They should be replaced by new ideas and methods of modern mechanics, mathematics, physics, and engineering sciences.

The approach proposed here deals with the construction of an adequate model of surface growth processes for solids (see also [3–9]). This approach is based on the following statements:

- We simulate the surface growth of a solid by the motion of its boundary due to the influx of new material to the surface of the solid.
- We obtain specific boundary conditions on the moving boundary (growth surface) as the result of an additional contact interaction problem between the 3D solid and the 2D surface, which depends on particular features of the growing process.
- We state the compatibility of the deformation rate tensor (or the stretching rate tensor) for a growing solid, while its strain tensor is, as a rule, incompatible.

The last statement leads to the case in which it is absolutely natural to choose the stress rate tensor, the deformation rate tensor, and the velocity vector as the basic variables in the governing system of equations describing the surface growth process.

In general, a boundary value problem for a growing solid contains three dependent controlled groups of values: the surface and bulk loadings, the tension of new adhering surfaces, and the velocity of influx of adhering surfaces.

If the velocity of the boundary particles of a growing solid is much less than the velocity of influx of new particles and the growth surface is closed, then the boundary value problem can be written in the form

$$\nabla\cdot\mathbf{S}=\mathbf{0},\quad \mathbf{D}=\frac{1}{2}\left[\nabla\mathbf{v}+(\nabla\mathbf{v})^{*}\right],\quad \mathbf{S}=G\left[2\mathbf{D}+(K-1)I_1(\mathbf{D})\mathbf{I}\right],$$

$$\mathbf{n}\cdot\mathbf{S}\big|_{\partial\Omega_1}=\mathbf{g},\quad \mathbf{v}\big|_{\partial\Omega_2}=\mathbf{v}_0,\quad \mathbf{n}\cdot\mathbf{S}\big|_{\partial\Omega(t)}=p\mathbf{n},$$

$$\mathbf{S}=\frac{\partial\mathbf{T}}{\partial t},\quad \mathbf{g}=\frac{\partial\mathbf{f}}{\partial t},\quad \mathbf{v}_0=\frac{\partial\mathbf{u}_0}{\partial t},\quad p=-s_n\left(\mathscr{T}_S:\mathscr{L}_S\right),\quad s_n=\mathbf{v}\cdot\mathbf{n}$$

where $\mathbf{T}$ is the stress tensor, $\mathbf{D}$ is the deformation rate tensor, $\mathbf{u}$ is the displacement, G and K are the elastic moduli, I_1 is the first invariant of a tensor, $\mathbf{I}$ is the unit tensor, $\mathbf{n}$ is the unit vector normal to the surface of the solid, $\mathbf{f}$ is the surface force, $\partial\Omega_1$ and $\partial\Omega_2$ are fixed domains of the solid surface, $\partial\Omega(t)$ is the growth surface, $\mathscr{T}_S$ is the 2D tensor of tension of the adhered surface, $\mathscr{Q}_S$ is the 2D tensor of curvature of the adhered surface, and the surface force on the growth surface, as well as the bulk force, is assumed to be zero.

One can readily see that the boundary value problem thus obtained has the same form as the classical boundary value problem of elasticity and that we can use all known analytical and numerical methods to solve this problem.

To obtain the stress and displacement values, one should use the formulas as follows (we denote by τ^* the time of adhesion of the surface to the growing solid and assume that the growth surface is always free of surface load):

$$\mathbf{T} = \mathbf{T}(\tau^*) + \int_{\tau^*}^{t} \mathbf{S} d\tau, \quad \mathbf{T}(\tau^*) = \mathscr{T}_S + (\mathbf{n} \cdot \mathbf{0})\mathbf{n} \otimes \mathbf{n},$$

$$\mathbf{u} = \mathbf{u}(\tau^*) + \int_{\tau^*}^{t} \mathbf{v} d\tau .$$

In the equations above, we have chosen the case of small deformations only to be definite. In the case of finite deformations, one should simply replace the linear Hooke's law by nonlinear constitutive relations.

This approach can be developed for the case of viscoelastic and aging materials by the methods in [10–12].

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