IUTAM Symposium on Growing solids
June 23-27, 2015, Moscow, Russia

On Compatibility Conditions for Propagating Surfaces with Additional Constraints

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The present report deals with the statement of boundary conditions on propagating wave surfaces of strong discontinuities in micropolar (MP) thermoelastic (TE) continua. An approach attributed to field theory is used to study the problem. A natural density of thermoelastic action and the corresponding variational least action principle are stated for a varying domain. A special form of the first variation of the action is employed to obtain 4-covariant jump conditions on the wave surfaces. These are given by the Piola–Kirchhoff stress 4-tensor and the energy–momentum tensor. The three-dimensional form of the jump conditions on the wave surfaces following from its four-dimensional covariant form is specified. These conditions should be supplemented by the geometrical and kinematical compatibility conditions due to Rankine, Hugoniot, Hadamard, and Thomas.

The general form of action in a variable region of a flat 4-spacetime with elementary volume $d^4X = dX^1 dX^2 dX^3 dX^4$ is

$$\mathfrak{I} = \int \mathsf{L}(X^\beta, \varphi^k, \partial_\alpha \varphi^k) d^4X \, , \tag{1}$$

where φ^k is the array of physical fields.

The least action principle says that the actual field is realized in the spacetime in such a way that the action (1) is minimal; i.e., for any admissible variations of the physical fields φ^k and the nonvariable coordinates X^β, one has $\delta\mathfrak{I} = 0$. Then the classical Euler–Lagrange equations hold, $\mathsf{E}_k(\mathsf{L}) = \dfrac{\partial \mathsf{L}}{\partial \varphi^k} - \partial_\beta \dfrac{\partial \mathsf{L}}{\partial(\partial_\beta \varphi^k)} = 0.$

The variation of action for *finite* variations of spacetime coordinates and physical fields can be represented in the form (see [1])

$$\delta\Im = \int \partial_\beta \left(T^{\beta\cdot}_{\cdot\alpha} \overline{\delta X^\alpha} - \underset{4}{S}{}^{\beta\cdot}_{\cdot k} \overline{\delta\phi^k} \right) d^4X,$$

where

$$\underset{4}{S}{}^{\beta\cdot}_{\cdot k} = -\frac{\partial \mathsf{L}}{\partial(\partial_\beta \varphi^k)}, \qquad T^{\beta\cdot}_{\cdot\alpha} = \mathsf{L}\delta^\beta_\alpha - (\partial_\alpha \varphi^k)\frac{\partial \mathsf{L}}{\partial(\partial_\beta \varphi^k)}.$$

The first variation of the action can be rewritten after applying the Gauss theorem in the form

$$\delta\Im = \oint_\partial \left(T^{\beta\cdot}_{\cdot\alpha} \overline{\delta X^\alpha} - \underset{4}{S}{}^{\beta\cdot}_{\cdot k} \overline{\delta\varphi^k} \right) \mathsf{N}_\beta d^3\tau. \qquad (2)$$

Here N_β is the normal 4-vector on the boundary surface.

Equation (2) can be further transformed if the field and coordinate variations on the variable boundary region are not independent. Thus variational formulation of the problem includes restrictions in the form of *imposed* boundary conditions on the variable integration surface of the domain such as $\varphi^k = \Gamma^k(X^\gamma)$ $(\gamma = \overline{1,4})$. Then the coordinate variations are coupled with the field variable variations as follows: $\delta\varphi^k = (\partial_\gamma \Gamma^k)\delta X^\gamma$ю,

Formula (2) becomes

$$\delta\Im = \oint_\partial \mathsf{N}_\beta \left(T^{\beta\cdot}_{\cdot\alpha} - (\partial_\alpha \Gamma^k) \underset{4}{S}{}^{\beta\cdot}_{\cdot k} \right) \delta X^\alpha d^3\tau.$$

Finally, in view of the arbitrariness and independence of the coordinate variations, the boundary conditions are obtained: $\mathsf{N}_\beta \left(T^{\beta\cdot}_{\cdot\alpha} - (\partial_\alpha \Gamma^k) \underset{4}{S}{}^{\beta\cdot}_{\cdot k} \right) = 0.$

The authors thank Professor Radayev for the statement of the problem and scientific advice.

The research was supported in part by the Russian Foundation for Basic Research (project No. 15-31-21111).

References

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