Some Problems in Mathematical Modeling of Biological Tissues Growth

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The present study presents some problems of mathematical modeling of surface growth of biological tissues. We state the fundamental principle of mathematical theory of surface growth in the case of large and small deformations. In particular, we discuss the characteristics of kinematic behavior of deformable solids for which the general tensor determining the kinematics is not the strain tensor or the strain rate tensor and the gradient rate tensor. We show that it is completely natural to consider the basic system of equations containing only velocities and stresses. If necessary, the displacements and stresses can be expressed by simple formulas via the initial values. Particular attention is paid to the statement of boundary conditions on the growth surface. The first condition is obtained by solving an auxiliary problem on the two-dimensional surface of contact with the three-dimensional growing solids. The second condition is the velocity of the growth surface. These conditions allow us to determine two relations between the three parameters: load, density, and the surface growth rate. One of these parameters may be predetermined, whereas the other two ones can be found by solving the boundary value problem. In some cases, this allows to control one of the parameters by the other two ones.

The main issue in modeling the growth of living tissues is the adequacy of the mathematical model and the ability to identify the constitutive constants from experiments. Therefore, the aim of the report is, in particular, to discuss the possibilities of using the proposed model as well as opportunities for experimental identification with a broad range of experts. Such problems have attracted much interest from mechanicians, mathematicians, and physicists. In this context, a number of problems associated with the effective provision of the necessary model parameters are considered. The problems of adequate description of the properties of hard and soft biological tissues using viscoelastic aging and anisotropic porous media are discussed.

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