Growing Elastic Hemisphere on a Smooth Rigid Foundation

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A growth process is a process of accreting material to the surface of a solid deformable body. The growth problem is the problem of determining the mechanical properties of a growing body. We consider the continuous growth (a thin layer of material is accreted to the body surface on a short time interval) of a hemisphere made of a linearly elastic material and its stress-strain state in the gravity field.

Assume that an arbitrary point O (Fig. 1) is selected in the plane and, for $t \in [0, t_1]$, some material is accreted to it under the action of centrally symmetric forces. The material forms a solid deformable

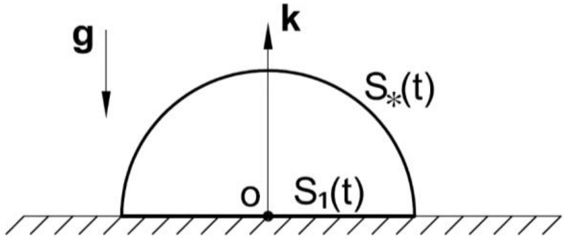

Figure 1: Growing hemisphere at time *t*.

hemisphere centered at point O. The gravity field is perpendicular to the plane, and the acceleration due to gravity is $\mathbf{g}$. We assume that the force of gravity acts only on particles that are part of the growing body and that there are no initial stresses in inflowing material.

At time t, the hemisphere occupies a region H_t of space with boundary $\partial H_t = S_*(t) \cup S_1(t)$, where $S_*(t)$ is the hemisphere to which additional material is accreted at this time and $S_1(t)$ is the boundary between the body and the plane. The particles on the surface $S_1(t)$ of the growing body form a smooth hard contact with the plane (Fig. 1).

We assume that the radius $R = R(t)$ of the hemisphere H_t is a smooth increasing function of time.

We are interested in the evolution of the stresses inside the hemisphere over time, assuming that the deformations of the particles are small and the deformation process is quasistatic.

The method for solving the problem is as follows. One transforms the original boundary value problem to a problem stated in velocities (the strain rate tensor and the partial time derivative of the stress tensor) and identical in the type of equations to the interior boundary value problem of the classical linear theory of elasticity. Then the problem in velocities is reduced to an equivalent problem, which is interpreted as a problem for a symmetrically loaded elastic sphere, whose solution is known [1] and can be represented by a series of Legendre functions. The solution of the original problem is finally obtained from the solution of the problem in velocities by integration with respect to time.

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