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Transient Temperature Fields in Growing Bodies Subject to Discrete and Continuous Growth Regimes

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A lot of natural phenomena and artificial processes are accompanied by deformation, non-uniform heating, and changing of the material composition of solids that flow simultaneously. Examples include formation of sedimentary, biogenic, volcanic rocks; crystallization, in particular snowflakes formation; electroplating, physical and chemical vapor deposition; solidification of melt; welding. From the mechanical point of view, such solids are classified as growing solids [1-3]. To determine the thermomechanical state of growing solids, one has to take into account the history of growth, or, better to say, the scenario of solid creation, even if locally the material is purely elastic (for example, belongs to the class of linearly thermoelastic materials). The mathematical description of this specific memory is manifested, in particular, by incompatible distortion fields and the corresponding residual stresses that cannot be removed by any smooth deformation. In this regard, mathematical models of growing bodies have much in common with models that appear in continual theory of defects. Such models require nonclassical methods for the statements of boundary value problems as well as for the construction of their solutions.

Unlike the general theory of growing solids, linear thermomechanical problems for growing solids go back a long way. One of them is the heat conduction problem with a moving boundary, which has been studied for more than 150 years. It was first stated by Lamé and Clapeyron in 1831. Notable research in this area is due to Joseph Stefan: he solved the problem while calculating how quickly an ice layer on water grows [4].

In the classical Stefan problem, the following conditions on the moving boundary (which separates the growing body and the environment) are stated: the temperature is continuous, whereas the heat flux has a discontinuity. The value of discontinuity is determined by the physical parameters of the formation of a new phase (for example, by the latent heat of crystallization). The generalized Stefan model assuming that the temperature at the interface is discontinuous as well was stated in [5]. This model, in particular, describes the dynamics of deposition processes in vacuum.

The subject of the present study is a growing linear thermoelastic deformable solid whose growth is due to the continuous flow (evaporation) of the material onto the boundary, with account of the fact that the temperature of the boundary differs from the temperature of the deposed material. We suppose that at the moment of joining a material particle (which, from the physical point of view, should be seen as a set of atomic-scale particles extensive enough to determine the thermodynamic variables) the temperature on the interface changes abruptly, causing an elementary thermal shock similar to one in the problem of V.I. Danilovskaya [6]. From this standpoint, we study a model problem for a parallelepiped whose material composition varies over time owing to the continuous joining of material to one of its faces. It is assumed that the growing surface being initially flat remains flat throughout the process of growth and moves progressive, generally, at a variable rate. Physically, this process corresponds to the idealized uniform deposition of material on a flat substrate. It should be noted that such theoretical studies were carried out for growing bodies of various canonical shape, in particular for a ball [7,8], but the statement of boundary conditions consistent with the thermomechanics of growth remained controversial. As a rule, it was assumed that the temperature on the growing boundary coincides with the temperature of the incoming material. (Here one can see an analogy with the classical Stefan problem.) However, a detailed study of nonstationary fields for the joining of a large number of discrete thin layers showed that the model with prescribed temperature on the boundary gives adequate physical description of the processes only under the condition that the characteristic time of the growing process is much greater than the characteristic time of relaxation of nonuniform thermal fields in the bulk of the growing solid [9]. In this regard, it is appropriate to consider the growing solid with discontinuous conditions for both the temperature and the heat flux (an analog of the generalized Stefan problem), the jump value being related to the characteristics of the growth process.

The statement of the boundary condition on the growing boundary with a detailed description of the physical (and chemical) processes seems to be extremely complicated and beyond the scope of this paper. The present results should be regarded as a rough approximation based on the following hypotheses.

- The joining material is layered; i.e., in an infinitesimal time, a layer of constant infinitesimal thickness is joined to the body.
- At the moment of joining, the material layer changes its temperature by a finite value during the infinitesimal time interval, thereby causing an infinitesimal heat shock on the growing boundary.
- The heat transfer with the environment does not occur.

The model based on these hypotheses can be considered as an idealization of a thin thermal barrier layer formed in a neighborhood the growth boundary.

Some assessment of the applicability of such a model can be given by comparing the stress and strain rate fields with the corresponding fields obtained in the framework of the model for discretely growing solids. To this end, we consider a sequence of such problems with increasing number of layers and decreasing thickness. However, it should be understood that a step function and a continuous function whose graphs are visually similar will never be fully equivalent to each other, and therefore, discrete growth and continuous growth have a qualitative difference (although maybe inessential for engineering applications).

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