Analysis of the Stress-Strain State of an Aging Viscoelastic Arch Fabricated Additively under Gravity for Various Time Modes of Layer-by-Layer Accretion

D. Parshin[1, 2] **and V. Hakobyan**[3]

[1] Institute for Problems in Mechanics of the Russian Academy of Sciences, Vernadsky Ave 101 Bldg 1, 119526, Moscow, Russia, parshin@ipmnet.ru

[2] Bauman Moscow State Technical University, 2nd Baumanskaya 5 Bldg 1, 105005, Moscow, Russia

[3] Institute of Mechanics, National Academy of Sciences of the Republic of Armenia, Yerevan, Armenia

The engineering problem of successive construction of a heavy circular vault (a cylindrical arch) on a smooth rigid horizontal base is studied (Fig. 1). The construction goes by layer-by-layer buildup of a

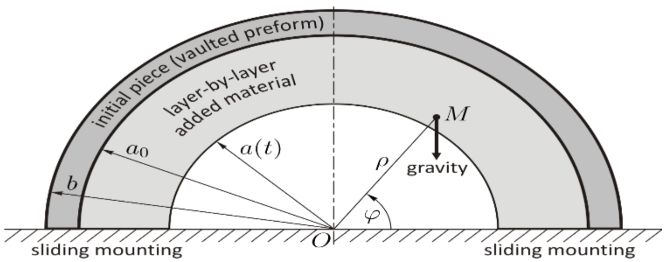

Figure 1: Scheme of the structure.

prefabricated residual stress-free vaulted preform originally installed on the base. We consider a buildup process in which a large number of thin layers of additional material are successively attached to the interior cylindrical surface of the arch. This process is modeled as a continuous accretion process; that is, we assume that an infinitely thin material layer is added to the solid on every infinitesimal time interval. The accreting material is assumed not to be prestressed; this means that the stresses are zero in the added material immediately upon attachment. We assume that fixation of the vault to the base prevents the vault feet separation from the base but permits their free sliding (the so-called “sliding mounting” of the vault). We examine the case of plane strain of the vault.

The material used in the construction (the additionally attached material as well as that of the initial piece) is assumed to be isotropic, uniformly aging, and viscoelastic (or elastic in particular case). We use a description of the material in the framework of the linear theory of creep of aging solids developed by Arutyunyan [1] and widely used when modeling the mechanical behavior of concrete (and also of rocks, soils, ice, and polymers).

The presented research is based on the fundamental concept of mechanics of growing solids, which was devised by Arutyunyan and his collaborators [2] and is now being actively developed in the mathematical theory of accreted solids by Manzhirov’s scientific school [3, 4].

We give statements of linear problems of mechanics that describe quasistatic deformation of the considered structure under the action of its own weight before and after the accretion process starts, including possible pauses between separate continuous accretion stages and an arbitrarily long time interval after the accretion process is complete. The statements of the problems that arise after the accretion process starts are nonclassical in deformable solid mechanics in that they do not involve a united natural (unstressed) configuration of the entire growing body, because there is no such configuration (in physical space) in principle. These problems are stated in terms of displacement rates and stress rates transformed by the viscoelasticity operator (so-called operator stresses) as initial–boundary value problems with specific boundary conditions on the considered body surface part, which continuously moves as additional material is accreted.

Analytical solutions of the problems stated are given by series and quadratures. We demonstrate the results of numerous numeral calculations performed for various time modes of the process under study both for the case of constructing thin- or thick-walled vaults using a thin-walled preform and for the case of reinforcement of originally thick-walled vaults. We show in a conclusive way that, by neglecting the gravity forces in the whole process of construction a heavy structure, one can get a dramatically wrong picture of both current and final states of the structure; in particular, one may overestimate the strength and the operational bearing capacity of the structure by a factor of several times. The effect of construction time mode on the deformation process of the vault accreted in the presence of gravity and under material creep and aging conditions is studied. We show that the construction time mode is critical for the stress-strain state of the heavy structures to be constructed. Moreover, the values of stress state characteristics in the course of the construction process may substantially exceed the steady-state values that onset after the completion of the process and may reach their maxima in other regions than the values in the final state do.

For example, a significant stress relief of the vault preform (initial part) material occurs at the expense of stressing the additional material due to large creeping resource if we provide a sufficiently high construction rate and use an early age material. However, an important feature of such a construction process is that the stress values in the vault preform at the initial stage of accretion exceed their values before the accretion. In the case of a thin-walled preform, the excess is highly substantial. If the construction process is slow, that is, if either the preform is installed on the base at a sufficiently old material age, or a prolonged pause before the start of its accretion is sustained, or the accretion process itself progresses too slowly, then the additional material attached in the final construction stage remains practically stress free. (Note that the extreme case of such processes, which is most representative for the manifestation of their peculiarities, is the solid construction using an elastic material.) In this situation, the stress level in the initial part of the structure increases greatly during its accretion.

We should add that the initial stresses in a thin-walled cylindrical heavy vault preform installed on a smooth rigid base are rather high. This guarantees that a ready-made structure constructed with the use of such a preform will contain regions where the stress level exceeds by a factor of several times the level predicted by the classical final configuration analysis, regardless of the structure construction rate. At the same time, for the case of a thin-walled preform, we show that if one varies the accretion rate in a suitable manner, then the stresses in the fabricated structure can eventually be diminished substantially relative to the initial preform state, so that the admissible stress value is not exceeded in the construction process. In the case of a sufficiently thick-walled preform, by varying the accretion rate one can obtain significantly lower stresses in the final-dimension structure than if it were installed on the base at once as a ready-made structure.

In the course of the analysis, we have also found that even an immensely thick-walled preform installed on a smooth base tends to separate from it in the peripheral feet regions due to gravity action. Without taking any special measures, it is impossible to get rid of the regions of separating contact stresses (negative contact pressure) on the constructed vault feet by the subsequent reinforcement of such a preform. This is true even if the final-dimension vault is sufficiently thick-walled for the contact pressure on its feet to be positive everywhere in case it were installed ready-made at once (rather than accreted).

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