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Quasi-Resonance Behavior of Viscoelastic Growing Rods Subjected to Free and Forced Longitudinal Vibrations

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A classical viscoelastic rod subjected to longitudinal vibrations was considered. The boundary conditions were such that the left end of the rod was fixed and right end was free. Moreover, it was assumed that the rod is growing and hence its length was changing in time. The function of growth was assumed to be twice continuously differentiable with respect to time. A particular case of rod's growth proportional to time was of special interest. The change of variables was introduced so that in new variables the rod length became constant. A new partial differential equation describing the rod dynamics was derived in these variables, which is as follows:

$$\frac{\partial^2 \bar{u}}{\partial \tau^2} - \frac{c^2}{\left[1+\varepsilon f(\tau)\right]^2}\left[1-\varepsilon^2 y^2 f'(\tau) - \frac{4\varepsilon\delta f'(\tau)}{1+\varepsilon f(\tau)}\right]\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{2\varepsilon y f'(\tau)}{1+\varepsilon f(\tau)}\frac{\partial^2 \bar{u}}{\partial \tau \partial y} - \frac{\varepsilon y f''(\tau)}{1+\varepsilon f(\tau)}\frac{\partial \bar{u}}{\partial y} \tag{1}$$
$$-\frac{2\delta}{\left[1+\varepsilon f(\tau)\right]^2}\frac{\partial^3 \bar{u}}{\partial \tau \partial y^2} + \frac{2\varepsilon^2 y f'^2(\tau)}{\left[1+\varepsilon f(\tau)\right]^2}\frac{\partial \bar{u}}{\partial y} + \frac{2\varepsilon\delta y f'(\tau)}{\left[1+\varepsilon f(\tau)\right]^3}\frac{\partial^3 \bar{u}}{\partial y^3} = F_0 \sin(\nu\tau)$$

This equation was simplified using the assumptions on slow rate growth constant and small viscoelastic damping factor. A special representation of the solution was introduced which used eigenfunctions of the generating problem, when growth and damping are neglected, and satisfying the boundary conditions. By means of this representation the governing partial differential equation was converted into an infinite system of ordinary differential equations. It was shown that the solutions of truncated systems converge to the solution of the original system of equations. Three major problems of the growing rod vibrations were stated and solved. In the first problem, free undamped vibrations were considered. It was shown that at linear growth of the rod the amplitudes of all its modes were also growing linearly in time. The simplified model neglecting the modes cross-coupling was composed for explanation of this effect. The corresponding differential equation was solved exactly in elementary functions and it was shown that the amplitudes of vibration of any modes grown linearly and almost-periods of vibrations had logarithmic dependence on time. In the second problem, free damped vibrations of linearly growing rod were considered. It was shown that time behavior of the rod has two characteristic domains: in the first, the vibration amplitudes decayed exponentially due to domination of the viscoelastic damping effects; in the second domain, these amplitudes started to grow linearly in time due to domination of the effects considered in the first problem (see the behavior of the first mode in Figure 1).

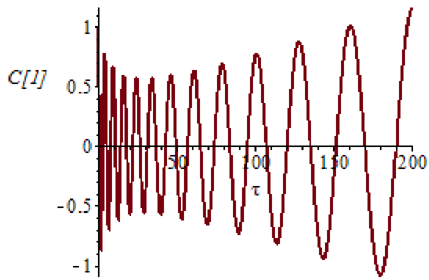

Figure 1: Quasi-resonance behavior of free growing viscoelastic rod.

The simplified model describing this effect and neglecting the modal cross-coupling was developed. The exact solution of the corresponding differential equation was obtained in the confluent hypergeometric functions, which qualitatively explained the abovementioned behavior of the rod. In the third problem, the forced damped vibrations of the rod were considered. It was shown that at fixed frequency of excitation the resonant effects were manifested subsequently at all modes of the rod in the process of its growth. The example of this behavior for the second mode is shown in Figure 2.

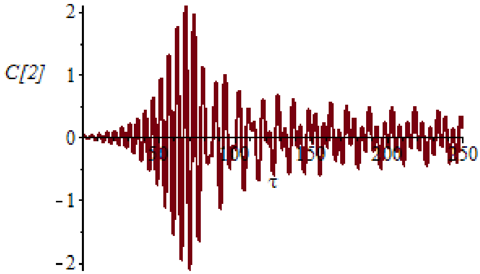

Figure 2: Quasi-resonance behavior of forced growing viscoelastic rod.

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